

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma(All)

Semester : 2

Date : 04/05/2017

Time : 02:00 To 05:00

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions:

(14)

- a) $|i + 3j - 2k| = \underline{\hspace{2cm}}$.
- (a) $\sqrt{14}$ (b) 14 (c) 2 (d) None of these
- b) If $(1, -2, 3) \cdot (4, 5, k) = 0$ then $k = \underline{\hspace{2cm}}$.
- (a) -2 (b) 2 (c) 1/2 (d) None of these
- c) If θ is the angle between the vectors \vec{x} and \vec{y} then $\sin\theta = \underline{\hspace{2cm}}$
- (a) $\frac{\vec{x} \times \vec{y}}{|\vec{x} \times \vec{y}|}$ (b) $\frac{|\vec{x} \times \vec{y}|}{|\vec{x}||\vec{y}|}$ (c) $\frac{\vec{x} \cdot \vec{y}}{|\vec{x} \times \vec{y}|}$ (d) None of these
- d) If $x = (1, 1, 1)$ and $y = (1, 0, 0)$ then $x - y = \underline{\hspace{2cm}}$.
- (a) (0,1,0) (b) (0,0,1) (c) (1,0,0) (d) None of these
- e) $\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \underline{\hspace{2cm}}$
- (a) m/n (b) n/m (c) 1 (d) None of these
- f) $\lim_{x \rightarrow 0} \left(1 + \frac{2}{x}\right)^x = \underline{\hspace{2cm}}$
- (a) e^2 (b) e (c) $e^{1/2}$ (d) None of these
- g) $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = \underline{\hspace{2cm}}$
- (a) ax^{a-1} (b) nx^{n-1} (c) na^{n-1} (d) None of these
- h) $\frac{d(x^2 + 2x + 7)}{dx} = \underline{\hspace{2cm}}$.
- (a) $x+2$ (b) $2x+2$ (c) 2 (d) None of these
- i) $\frac{d(\sec^2 x - \tan^2 x)}{dx} = \underline{\hspace{2cm}}$.
- (a) 2 (b) 1 (c) 0 (d) None of these



- j) $\frac{d(6^x)}{dx} = \underline{\hspace{2cm}}$
 (a) $6^x \log_e 6$ (b) $\log_e 6$ (c) $x^6 \log_e 6$ (d) $6^x \log_e x$
- k) If $f(x) = \log \sqrt{x^2 + 1}$ then $f'(0) = \underline{\hspace{2cm}}$
 (a) $\log 2$ (b) $\frac{1}{2} \log 2$ (c) $2 \log 2$ (d) 0
- l) $\int \frac{-1}{x^2 + 1} dx = \underline{\hspace{2cm}}$
 (a) $\tan^{-1} x + c$ (b) $\sin^{-1} x + c$ (c) $\cos^{-1} x + c$ (d) $\cot^{-1} x + c$
- m) $\int \frac{1}{\sqrt{x^2 + 4}} dx = \underline{\hspace{2cm}}$
 (a) $\cot^{-1} \frac{x}{2} + c$ (b) $\tan^{-1} \frac{x}{2} + c$ (c) $\log \left| x + \sqrt{x^2 + 4} \right| + c$ (d) none of these
- n) $\int_4^5 \frac{1}{x} dx = \underline{\hspace{2cm}}$
 (a) $\log \frac{4}{5}$ (b) $\log \frac{5}{4}$ (c) $\log 10$ (d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) If $\bar{a} = i + 2j - k$, $\bar{b} = 3i + j + 2k$ and $\bar{c} = -2i - j + 5k$ then find $|2\bar{a} + 3\bar{b} - \bar{c}|$. (5)
- b) If $\bar{a} = i + j + k$ and $\bar{b} = 2i - 2j + k$ then find unit vector perpendicular to \bar{a} and \bar{b} . (5)
- c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$ (4)

Q-3 Attempt all questions (14)

- a) Prove that $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x} = \frac{1}{\sqrt{a}}$. (5)
- b) Evaluate: $\lim_{x \rightarrow 0} \frac{2(5^x) + 3(2^x) - 5}{x}$ (5)
- c) Find $\frac{dy}{dx}$ if $y = \frac{a + b \sin x}{a \sin x + b}$ (4)

Q-4 Attempt all questions (14)

- a) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$. Find velocity at $t = 0$ and acceleration at $t = 1$. (5)
- b) Find $\frac{dy}{dx}$ if $y = \log \left(\frac{\sin x}{1 + \cos x} \right)$. (5)
- c) For what value of m , vectors $m\bar{j} + 2\bar{i} + \bar{k}$ and $2\bar{i} + 4\bar{j} + 5\bar{k}$ are perpendicular (4)



to each other?

Q-5 Attempt all questions (14)

a) Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$. (5)

b) Find $\frac{dy}{dx}$ if $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$. (5)

c) Evaluate: $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$ (4)

Q-6 Attempt all questions (14)

a) Forces $\vec{F}_1 = i + 2j - 3k$ and $\vec{F}_2 = i - j + 2k$ act on a particle under the influence of these forces, particle moves from point $(3, 1, 2)$ to $(1, 3, -1)$. Find the work done. (5)

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$ (5)

c) Find derivative of $f(x) = e^x$ using definition. (4)

Q-7 Attempt all questions (14)

a) Evaluate: $\int x \sin x dx$ (5)

b) Find area of region bounded between $y = x^2$, X-axis, $x = 1$ and $x = 2$. (5)

c) If $\vec{a} = (2, -3, -1)$ and $\vec{b} = (1, 4, -3)$ then find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. (4)

Q-8 Attempt all questions (14)

a) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. (5)

b) Find $\frac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. (5)

c) Evaluate: $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$ (4)

